Rejecting Small Gambles Under Expected Utility*

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Abstract

Rejecting a gamble over a given range of wealth levels imposes a lower bound on risk aversion. Using this lower bound on risk aversion and empirical evidence on the range of the risk aversion coefficient, we calibrate the relationship between risk attitudes over small-stakes and large-stakes gambles. We find that rejecting small gambles is consistent with expected utility, contrary to a recent literature that concludes that expected utility is fundamentally unfit to explain decisions under uncertainty. Paradoxical behavior is only obtained when calibrations are made in a region of the parameter space that is not empirically relevant.

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"The report of my death was an exaggeration."
Mark Twain (May 1897)

1 Introduction

Within expected utility theory, risk aversion is identified with the concavity of the Bernoulli utility function u on wealth w (Arrow (1971), Pratt (1964); see Mas-Colell et al. (1995, chapter 6)). The expected utility framework has been severely criticized in a recent literature that concludes that diminishing marginal utility is an utterly implausible explanation for appreciable risk aversion.[1] The basis of the criticism can be best illustrated in Rabin (2000) who calibrates the relationship between risk attitudes over small and large stakes gambles under expected utility. Using calibration results, it is possible to present striking statements of the following kind: if a decision maker is a risk-averse expected utility maximizer and if he rejects gambles involving small stakes over a large range of wealth levels, then he will also reject gambles involving large stakes, sometimes with infinite positive returns. For instance, "suppose that, from any initial wealth level, a person turns down gambles where she loses \$100 or gains \$110, each with 50% probability. Then she will turn down 50-50 bets of losing \$1,000 or gaining *any* sum of money," or "suppose we knew a risk averse person turns down 50-50 lose \$100/ gain \$105 bets for any lifetime wealth level less than \$350,000 . . . Then we know that from an initial wealth level of \$340,000 the person will turn down a 50-50 bet of losing \$4,000 and gaining \$635,670." (Rabin (2000, p. 1282)).

¿From this paradoxical, even absurd, behavior towards large-stakes gambles, Rabin (2000) and other authors conclude that expected utility is fundamentally unfit to explain decisions under uncertainty. This paper challenges this conclusion. We show that the flaw identified in this literature has little empirical support. In particular, we show that it is the

[1] See, for example, Hansson (1988), Kandel and Stambaugh (1991), Rabin (2001), Rabin and Thaler (2001) and other references therein. Samuelson (1963), Machina (1982), Segal and Spivak (1990) and Epstein (1992) also study various issues that are related to this literature.

assumption of rejecting small gambles over a large range of wealth levels, and *not* expected utility, the one that does not typically match real-world behavior.

The plausibility of the "rejecting small gambles" assumption is argued in the literature purely by appealing to the reader's introspection. Introspection, however, may sometimes be misleading. Indeed, we shall see that the assumption is far from being tautological or empirically compelling. Rather than relying on introspection, we investigate the empirical implications of this assumption on the preferences of the decision maker. We shall demonstrate that it implies a specific positive lower bound on the coefficient of absolute risk aversion, and show how it can be calculated. Indeed, over the relevant range of wealth levels, something beyond strict concavity is being assumed.[2] The relevant question is then to determine how high is the implied lower bound on risk aversion, that is whether this bound is broadly consistent with the shape of the utility functions supported by empirical evidence. We argue that this bound is often unreasonably high. For instance, the assumption that a person turns down gambles where she loses \$100 or gains \$110 for *any* initial wealth level implies that the coefficient of relative risk aversion must go to infinity when wealth goes to infinity, while the assumption that a 50-50 lose \$100/ gain \$105 bets is turned down for any lifetime wealth level less than \$350,000 implies a value of the same coefficient no less than 166.6 at \$350,000. In contrast, a vast body of empirical evidence consistently indicates that the coefficient of relative risk aversion is estimated to be in the single-digit range. With this range of empirically plausible values in hand, we calibrate the relationship between risk attitudes over small-stakes and large-stakes gambles. We do not find paradoxical rejections of large-stakes gambles. Paradoxical behavior is only obtained when calibrations are made in a region of the parameter space that is not empirically relevant.

[2]This is contrary to what is claimed in the literature. For instance, "the calibration theorem is entirely nonparametric, assuming *nothing* about the utility function except concavity" (Rabin (2000, p. 1282)). However, we show that there are large families of concave functions that are being ruled out by this assumption.

2 Rejecting Small Gambles: Testable Implication and Some Calibrations

As explained for example in Segal and Spivak (1990), any expected utility agent with a differentiable utility function must accept infinitesimal gambles of positive expected value, because locally these preferences amount to risk neutrality. However, as soon as the gambles are no longer infinitesimal, but of any finite size, expected utility is compatible with both accepting and rejecting small gambles. It is then necessary to perform more powerful tests to examine the question of the appropriateness of expected utility preferences.

There is much work, both experimental and empirical, concerning the behavior of agents towards small-stakes gambles under expected utility. Camerer (1995) provides a comprehensive survey of experimental evidence on individual decision making, virtually gathered in all cases from what may be considered small-stakes gambles. He finds substantial support for expected utility. In particular, he concludes that expected utility appears to be a helpful model to understand the reactions of individuals to risk observed in experimental data involving small gambles, and that expected utility theory may be considered in the efficient frontier of available theories.[3] As to the range of risk aversion estimates obtained in experimental studies of games, the evidence is quite robust. Goeree, Holt, and Palfrey (2000, 2001), for instance, examine several asymmetric matching pennies games and private-values auction experiments, respectively. These experiments involve very small gambles, and total payoffs after all rounds have been completed typically range from 5 to

[3]For instance, after evaluating the relative performance of different theories, he concludes that "the result is an 'efficient frontier' of theories that are more accurate, best fitting, given the number of patterns they allow. A compilation of 23 data sets from a total of 2,000 subjects making 8,000 choices shows that the following theories are on the efficient frontier: mixed fanning, prospect theory, expected utility and expected value" (p. 638). He also concludes "that many subjects obey expected utility and that the lean functional form of expected utility is more statistically robust to estimation error than more complex functional forms" (p. 640), and that "more general theories appear to fit better than expected utility since they have more degrees of freedom, but are not better in predicting new choices" (p. 642). See also Loomes and Segal (1994) for an experimental study that provides evidence of different orders of risk aversion, including local risk neutrality.

20 dollars per individual subject. Their estimates of the relative risk aversion coefficient are virtually in all cases below 1, and highly significant across treatments and games: estimates typically range from 0.3 to 0.7, centered around 0.5. The value of 0.5 is also almost identical to that obtained in many experimental studies of similar nature that these authors cite.

The empirical evidence on decisions under uncertainty in natural environments is vast, and not easy to summarize. However, the relevant conclusion for the purposes of our analysis is again noticeably robust: coefficients of relative risk aversion, $r_R(w, u)$, in the single-digit range appear to rationalize the reactions to risk in virtually all circumstances examined in the literature. Indeed, the evidence on the size of this coefficient is remarkably consistent across hundreds of studies that examine the behavior of agents choosing among risky alternatives that greatly differ in the scale of risk in a broad variety of economic contexts.[4]

It is important to remark that the robust conclusion that is obtained regarding the empirical values of $r_R(w, u)$ includes many studies that involve gambles of a "small" magnitude. These include studies that examine the behavior of individuals facing familiar gambles such as risk-dollar tradeoffs for minor health hazards in the use of insecticides and toilet bowl cleaner or the choice between alternative local telephone calling plans.[5] The results in these studies show that many agents do accept small-stakes gambles. Further, they yield low estimates of the relative risk aversion coefficient, and often even support the

[4]For example, Mehra and Prescott (1985) review several microeconometric studies and consider that the evidence "constitute an a priori justification for restricting the value of this coefficient to be a maximum of ten," as they do in their study. Kocherlakotta (1996) concludes from his review of financial markets that "a vast majority of economists believe that values above 10 (or for that matter above 5) imply highly implausible behavior on the part of individuals." Indeed, Luttmer (1996) shows how single-digit values of $r_R(w, u)$ can reconcile the size of the equity premium taking into account actual capital market frictions. See also Rosenzweig and Wolpin (1993), Barsky et al. (1997), and Binswanger (1980) for other relevant studies and further references.

[5]See Evans and Viscusi (1991) and Miravete (2001). The magnitudes of the gambles in these studies would certainly appear to be negligible for *any* consumer. Likewise, evidence on risk attitudes from natural experiments involving television game shows estimate $r_R(w, u)$ to be in the single-digit range and often cannot reject the null hypothesis that agents are risk neutral (see, for example, Metrick (1995)).

hypotheses of risk neutrality and risk loving behavior for small scale risks –in this respect, the empirical evidence concerning individuals who purchase lottery tickets or engage in similar forms of gambling is also extensive.

We thus consider that the experimental and empirical evidence conclusively indicates the range of parameter values where it would be empirically relevant to calibrate attitudes towards risk for gambles of different size.[6]

Rabin (2000) shows that if an individual is a risk averse expected utility maximizer and rejects a given gamble of equally likely gain g and loss l, $g > l$, over a given range of wealth levels, then he will reject correspondingly larger gambles of gain G and loss L. We investigate now the implication of the "rejecting small gambles" assumption.

For a decision maker with wealth level w and twice continuously differentiable Bernoulli utility function u, denote the Arrow-Pratt coefficient of absolute risk aversion by $r_A(w, u) = -\frac{u''(w)}{u'(w)}$, with $r_R(w, u) = w \cdot r_A(w, u)$. We next show that the assumption that an expected utility maximizer turns down a given gamble with gain g and loss l for a given range of wealth levels implies that there exists a positive lower bound on $r_A(w, u)$. In fact, this positive lower bound can be calculated exactly and, therefore, provides a testable implication of the assumption.

Proposition. Let u satisfy non-increasing absolute risk aversion. Let I be an interval in the positive real line. If for every $w \in I$,

$$\frac{1}{2}u(w+g) + \frac{1}{2}u(w-l) < u(w),$$

[6] A final observation concerning the evidence is in order. While the theory usually focuses on life-time wealth, empirical studies typically use per period data (e.g., monthly income, yearly consumption, and so on). Under the standard assumption of time stationarity of preferences, one can easily show that estimates of the preference parameters for the per period utility function correspond to those of the utility function over life-time wealth. Also, Rubinstein (2001) notes that "nothing in the von Neumann-Morgenstern (vNM) axioms dictates use of final wealth levels ... vNM are silent about the definition of prizes ... The definition of prizes as final wealth levels is no less crucial to Rabin's argument than the expected utility assumption." We will thus calibrate attitudes towards risk for gambles of different size in the region where the combination of prizes (which we will continue referring as wealth) and the curvature of the utility function yield empirically plausible parameter values.

there exists $a^* > 0$ such that the absolute risk aversion coefficient $r_A(w, u)$ is greater than a^* for all $w \in I$. Moreover, the highest such a^* is the solution to the equation

$$f(a) = e^{al} + e^{-ag} - 2 = 0.$$

Proof: Suppose not. Then, for every $a > 0$ there exists $w \in I$ such that $r_A(w, u) < a$. In particular, this holds for the unique $a^* > 0$ solving the equation $f(a) = 0$. (By the intermediate value theorem, note that a^* is well defined because f is continuous, $f(0) = 0$, $f(\infty) = \infty$, $f'(0) < 0$, $f'(a) = le^{al} - ge^{-ag}$ is first negative, vanishes at a single point and is positive thereafter).

Consider the constant absolute risk aversion utility function $v(w) = -e^{-aw}$ for $a < a^*$. For such a choice of a, $f(a) < 0$, i.e.,

$$e^{al} + e^{-ag} < 2,$$

or

$$\frac{1}{2}\left(-e^{-a(w-l)}\right) + \frac{1}{2}\left(-e^{-a(w+g)}\right) > -e^{-aw}.$$

Thus, an individual with utility function v would agree to play the small-stakes lottery with gain g and loss l starting from any wealth level w.

Denote by $w' \in I$ the wealth level for which $r_A(w', u) = a$. By non-increasing absolute risk aversion, for $w \in I$, $w \geq w'$, the individual with utility function v is at least as risk averse as the one with utility function u. Therefore, using the well-known characterization of comparisons of risk attitudes across individuals in Rothschild and Stiglitz (1970), it follows that

$$\frac{1}{2}u\left(w+g\right) + \frac{1}{2}u\left(w-l\right) > u(w)$$

for every $w \in I$, $w \geq w'$, which is a contradiction. ■

The hypothesis of non-increasing absolute risk aversion seems to be generally accepted. Note, however, that it is not essential to the argument in the above proof. It is used only in its last step to assert that the range of wealth levels over which the "rejecting the small-stakes lottery" assumption is violated is an interval.

Hence, contrary to several statements in Rabin (2000, 2001) and Rabin and Thaler (2001), the conclusion to be drawn from this proposition is that the assumption of rejecting the small-stakes gamble does go beyond the assumption of concavity of the Bernoulli utility function. A positive lower bound on risk aversion is also assumed, and this bound is independent of the interval I over which the assumption is made. This lower bound on the coefficient of relative risk aversion clearly increases when for a given small-stakes gamble we enlarge the interval I over which it should be rejected. This means that the assumption that a given gamble is rejected for all wealth levels is incompatible with the agent becoming risk neutral at some sufficiently high level of wealth, feature shared by a large class of concave utility functions. The proposition implies that $r_R(w, u)$ must go to infinity as wealth goes to infinity.

Next, having turned the assumption of rejecting small gambles into an empirically testable proposition we examine whether or not rejecting small gambles in the empirically relevant parameter space, that is when the coefficient $r_R(w, u)$ is in the single-digit range, induces paradoxical behavior. Although the point we are raising is general, only for computational simplicity, we shall work with the class of CRRA (constant relative risk aversion) Bernoulli utility functions $u(w) = \frac{w^{1-\gamma}}{1-\gamma}$ for $\gamma \geq 0$.[7] For this utility function, $r_R(w, u) = \gamma$. In order to facilitate comparison with the literature we next consider calibrations based on gambles similar to Rabin (2000). This is the only reason to use these gambles, in the absence of a clear definition of what it is a "small gamble." One can certainly argue that there is a great deal of subjectivity in evaluating the magnitude of a gamble.[8]

The assumption of rejecting small-stakes gambles is generally made over a given range of wealth levels. In Table I we calculate, for two small-stakes lotteries and for different values of γ, the largest wealth level at which an individual rejects the lotteries.

[7]Our arguments in what follows are unaffected if we make the calculations using more general specifications.

[8]If one is to assume the rejection of an initial small gamble, this should hold with independence of how it is presented (e.g., as the choice between telephone calling plans, as a minor health hazard, or as a choice in a first-price auction or a matching pennies experiment).

[Table I here]

Note that the values of these wealth levels are extremely small. Therefore, the empirical relevance of the assumption, for decision makers with γ in the single digits, would seem to be quite limited.

Continuing with the specification of CRRA utility, the next question we examine is how high is the bound a^* associated with the given small-stakes lottery of gain g and loss l. On the basis of the same lotteries used in Rabin's (2000) Tables I and II, we calculate in Table II their corresponding values of a^*, as defined in the proposition above. The table also shows, for wealth levels of \$300,000 and \$30,000, the induced values of γ.

[Table II here]

It is first worth noting that for the wealth level of \$300,000 very few values of γ are in the single-digit range, or even in the teens. No single-digit value arises when the gamble involves losing \$100 or \$1,000. Only when the rejected gamble involves losing $l = \$10,000$, which would not appear to be a small-stakes gamble, such low values start to arise consistently. In an attempt to generate more γ coefficients in the single-digit range, we examine the same lotteries for a wealth level as extremely low as \$30,000. In this case, single-digit coefficients arise for some gambles where $l = \$1,000$, and for all gambles where $l = \$10,000$, which are hardly small-stake gambles for an individual with that wealth level. For the lowest stakes gambles involving $l = \$100$ a single-digit γ is only found when $g = 101$. We conclude from Table II that empirically plausible, single-digit values of γ are compatible with the assumption only when the loss l in the gamble is a significant proportion of the individual's wealth. We thus learn that the assumption of rejecting truly small gambles does not hold, when applied to all the decision makers that are behind the experimental and empirical evidence mentioned above.

Finally, for various lotteries in Table II that yield values of γ in the single-digit range, Table III displays the best large-stakes lottery with gain G and loss L that the decision maker would reject.

[Table III here]

It is apparent that these rejections are no longer paradoxical. For instance, for a wealth of \$300,000 the agent turns down gambles involving losses L ranging from 5 to 15 percent of his wealth and gains G that appear reasonable. The same can be said for a wealth of \$30,000. In this case, note that relative to wealth these values of L are ten times greater than those in Rabin (2000). Thus, not even for these much larger gambles paradoxical behavior is obtained. Finally, it is worth stressing that gambles with $G = \infty$ are turned down only when potential losses L represent a significantly great proportion of the individual's wealth.

The reasonable behavior described in these large-stakes gambles contrasts with the paradoxes in Rabin (2000) and in other authors in the literature, and indeed may be viewed as a further confirmation of the empirical soundness of single-digit values for $r_R(w, u)$.

These results refute assertions such as "paradoxical implications are not restricted to particular contexts or particular utility functions," or "within the expected-utility framework, for *any* concave utility function, even very little risk aversion over modest stakes implies an absurd degree of risk aversion over large stakes" (Rabin (2001, p. 203)). That is, much more than "very little risk aversion over modest stakes" is needed to generate paradoxical behavior. Indeed, this is only obtained when calibrations are made in a region of the parameter space that is not empirically relevant.

Lastly, it is important to note that a rather straightforward empirical implication of the calibrations in the region of the parameter space considered in the literature is that "when measuring risk attitudes maintaining the expected-utility hypothesis ... data sets dominated by modest-risk investment opportunities are likely to yield much higher estimates of risk aversion than data sets dominated by larger investment opportunities" (Rabin (2000, p. 1287)). Contrary to this implication, the empirical evidence gathered from many different studies consistently obtains estimates of $r_R(w, u)$ that vary very little across a wide heterogeneity of scales of risk, as these estimates are narrowly confined to the single-digit range.

3 Concluding Remarks

Using a problem posed to one of his colleagues as a starting point, Samuelson (1963) argues that, under expected utility, the rejection of a given single gamble for all wealth levels implies the rejection of the compound lottery consisting of the single gamble being repeated an arbitrary number of times. Samuelson's exercise sheds light on the fact that some decision makers may be misapplying the law of large numbers when accepting a compound lottery (the colleague's response was that he would reject the single lottery, but accept its compound version). However, Samuelson was clearly aware of the crucial importance of the assumption of rejecting the single lottery for *all* wealth levels or a large range thereof: "I should warn against undue extrapolation of my theorem. It does not say that one must always refuse a sequence if one refuses a single venture: if, at higher income levels the single tosses become acceptable, and at lower levels the penalty of losses does not become infinite, there might well be a long sequence that it is optimal" (p. 112). Indeed, it may very well be the case that Samuelson's colleague was not fooled by any fallacy of large numbers. He simply violated the assumption of rejecting the given small-stakes lottery for all wealth levels or large range thereof.

The main advantage of expected utility is its simplicity and its usefulness in the analysis of economic problems involving uncertainty. As often argued in the literature, its predictions sometimes conflict with people's behavior. This has led economists to develop various non-expected utility models which can often accommodate actual behavior better. The non-expected utility research agenda is an important one, and there is no question that we should continue to pursue it. However, expected utility should not be accused when it is not at fault. The analysis in this paper shows how certain paradoxical examples in the literature are many times counterfactuals. Paradoxical behavior is only obtained when calibrations are made in a region of the parameter space that is not empirically relevant. In a more recent paper, Rabin and Thaler (2001) continue to drive home the theme of the demise of expected utility and compare expected utility to a dead parrot from a Monty Python show. To the extent that all their arguments are based on the calibrations in Rabin

(2000), the expected utility parrot may well be saying that "the report of my death was an exaggeration."

REFERENCES

ARROW, K. J. *Essays in the Theory of Risk Bearing.* Chicago, IL: Markham Publishing Company, 1971.

BARSKY, R. B., F. J. JUSTER, M. S. KIMBALL, AND M. D. SHAPIRO. "Preference Parameters and Behavioral Heterogeneity: An Experimental Approach in the Health and Retirement Study." *Quarterly Journal of Economics* 112 (1997): 537-579.

BINSWANGER, H. P. "Attitude Toward Risk: Experimental Measurement in Rural India." *American Journal of Agricultural Economics* 62 (1980): 395-407.

CAMERER, C. "Individual Decision Making." In J. Kagel and A. Roth (eds.) *Handbook of Experimental Economics*, Princeton University Press, 1995.

EPSTEIN, L. G. "Behavior under Risk: Recent Developments in Theory and Applications." In J. J. Laffont (ed.) *Advances in Economic Theory. VI World Congress of the Econometric Society (vol. II)*, Cambridge University Press, 1992.

EVANS, W. N. AND W. K. VISCUSI. "Estimation of State-Dependent Utility Functions Using Survey Data." *Review of Economics and Statistics* 73 (1991): 94-104.

GOEREE, J. K., C. A. HOLT AND T. R. PALFREY. "Risk Averse Behavior in Asymmetric Matching Pennies Games." Mimeo (2000), Caltech.

GOEREE, J. K., C. A. HOLT AND T. R. PALFREY. "Quantal Response Equilibrium and Overbidding in Private-Value Auctions." *Journal of Economic Theory* (2001), forthcoming.

HANSSON, B. "Risk Aversion as a Problem of Cojoint Measurement." In *Decision, Probability, and Utility*, ed. by P. Gardenfors and N.-E. Sahlin. Cambridge: Cambridge University Press, 1988.

KANDEL, S., AND R. F. STAMBAUGH. "Asset Returns, Investment Horizon, and Intertemporal Preferences." *Journal of Monetary Economics* 27 (1991): 39-71.

KOCHERLAKOTTA, N. R. "The Equity Premium: It's Still a Puzzle." *Journal of Economic Literature* 34 (1996): 42-71.

LOOMES, G. AND U. SEGAL. "Observing Different Levels of Risk Aversion." *Journal of Risk and Uncertainty* 9 (1994): 239-256.

LUTTMER, E. G. J. "Asset Pricing in Economies with Frictions." *Econometrica* 64 (1996): 1439-1467.

Machina, M. "Expected Utility Analysis without the Independence Axiom." *Econometrica* 50 (1982): 277-323.

Mas-Colell, A., M. D. Whinston and J. R. Green. *Microeconomic Theory*, Oxford University Press, 1995.

Mehra, R. and E. C. Prescott. "The Equity Premium: A Puzzle." *Journal of Monetary Economics* 15 (1985): 145-161.

Metrick, A. "A Natural Experiment in "Jeopardy!" *American Economic Review* 85 (1995): 240-53.

Miravete, E. J. "Choosing the Wrong Calling Plan? Ignorance, Learning, and Risk Aversion." Mimeo, Dept. of Economics, University of Pennsylvania, 2001.

Pratt, J. "Risk Aversion in the Small and in the Large." *Econometrica* 32 (1964): 122-136.

Rabin, M. "Risk Aversion and Expected-Utility: A Calibration Theorem." *Econometrica* 68 (2000): 1281-1292.

Rabin, M. "Diminishing Marginal Utility of Wealth Cannot Explain Risk Aversion." In *Choices, Values, and Frames.* D. Kahneman and A. Tversky, eds. Cambirdge University Press, 2001.

Rabin, M. and R. H. Thaler. "Anomalies. Risk Aversion." *Journal of Economic Perspectives* 15 (2001): 219-232.

Rothschild, M. and J. E. Stiglitz. "Increasing Risk: I. A Definition." *Journal of Economic Theory* 2 (1970): 225-243.

Rosenzweig, M. R. and K. I. Wolpin. "Credit Market Constraints, Consumption Smoothing, and the Accumulation of Durable Assets in Low-Income Countries: Investments in Bullocks in India." *Journal of Political Economy* 101 (1993): 223-244.

Rubinstein, A. "Comments on the Risk and Time Preferences in Economics." Mimeo (2001), Tel Aviv and Princeton University.

Samuelson, P. A. "Risk and Uncertainty: A Fallacy of Large Numbers." *Scientia* 98 (1963): 108-113.

Segal, U. and A. Spivak. "First Order versus Second Order Risk Aversion." *Journal of Economic Theory* 51 (1990): 111-125.

TABLE I

Wealth levels at which an individual with CRRA (γ) utility function stops rejecting a 50-50 lose $100 / gain $*g* lottery.

	g	
γ	125	110
2	400	1,000
3	1,501.3	3,300.5
4	2,003.1	4,401.2
5	2,504.9	5,502
6	3,006.9	6,602.7
7	3,508.8	7,703.5
8	4,010.8	8,804.3
9	4,512.8	9,905.1
10	5,014.9	11,006
11	5,516.9	12,106.8
12	6,018.9	13,207.6
20	10,035.4	22.014.2
30	15,056	33,022.5
40	20,076.7	44,030.8
50	25,097.3	55,039.2

TABLE II

Lower bounds on the coefficient of absolute risk aversion for an individual that rejects a 50-50 lottery lose $\$l$ / gain $\$g$ (a*) for any range of wealth levels, and associated lower bound on the coefficient of relative risk aversion for wealth levels $300,000 and $30,000.

$l\,/\,g$	a*	γ = 300,000a*	γ = 30,000a*
100 / 101	.0000990	29.7	2.9
100 / 105	.0004760	142.8	14.2
100 / 110	.0009084	272.5	27.2
100 / 125	.0019917	597.5	59.7
100 / 150	.0032886	986.5	98.6
1,000 / 1,050	.0000476	14.2	1.4
1,000 / 1,100	.0000908	27.2	2.7
1,000 / 1,200	.0001662	49.8	4.9
1,000 / 1,500	.0003288	98.6	9.8
1,000 / 2,000	.0004812	144.3	14.4
10,000 / 11,000	.0000090	2.7	0.2
10,000 / 12,000	.0000166	4.9	0.4
10,000 / 15,000	.0000328	9.8	0.9
10,000 / 20,000	.0000481	14.4	1.4

TABLE III

If averse to 50-50 lose \$*l* / gain \$*g* for wealth levels \$300,000 and \$30,000 with CRRA utility and coefficient of relative risk aversion γ, will also turn down a 50-50 lose *L* / gain *G* bet; *G*'s entered in Table.

Wealth: \$30,000

l/g:	100/101	1,000/1,050	1,000/1,100
γ:	2.9	1.4	2.7
L			
400	416	---	---
600	636	---	---
800	867	---	---
1,000	1,107	---	---
2,000	2,479	2,205	2,439
4,000	6,538	4,917	6,259
6,000	14,538	8,329	13,168
8,000	40,489	12,749	30,239
10,000	∞	18,686	495,319

Wealth: \$300,000

l/g:	10,000/11,000	10,000/12,000
γ:	2.7	4.9
L		
15,000	17,341	19,887
17,000	20,072	23,572
20,000	24,393	29,792
22,000	27,435	34,490
25,000	32,266	42,574
30,000	41,116	59,870
40,000	62,594	126,890
50,000	91,268	∞
75,000	239,089	∞