Can more police induce more crime?*

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Abstract

We show that for any appropriation technology available to thieves, it is possible to incorporate it into a Walrasian economy so that in equilibrium more police induces more crime. Moreover, this perverse effect could arise even if the level of police protection is the socially optimal one.

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1 Introduction

One of the more longstanding questions in the empirical literature on crime concerns the extent to which police affects crime.[1] It would be no exaggeration to say that

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[1] Prominent exponents of this literature are Marvell and Moody (1996), Levitt (1997, 2002), Evans and Owens (2007), Di Tella and Schargrodsky (2004), Klick and Tabarrok (2005), Draca,

most of the papers in this literature open by claiming that previous studies have not found any causal effect of police on crime, the reason being some flaw in the data or in the econometric analysis. These papers then attempt to approach the question in a way that is immune to these flaws and indeed provide some reliable evidence of the causal effect of police on crime. Despite many efforts, it is safe to say that this literature has not yet provided a definitive answer.

Another characteristic of the empirical literature on crime and police is that it seems to stand on quite modest theoretical foundations. As Burdett, Lagos, and Wright (2003) aptly say, "much (although not all) work on the economics of crime uses partial equilibrium reasoning or empirical methods with very little grounding in economic theory." Furthermore, most of the empirical papers that investigate the causal effect of police on crime do not explicitly formulate the structural model on which they are based, making it difficult to figure out whether they estimate a reduced form model or a parameter of some structural equation.

One of the possible reasons for this meager theoretical support could be that models of crime are somewhat scarce. Indeed, Polinsky and Shavell (2000) and Chalfin and McCrary (2017) are two recent surveys on law enforcement and deterrence which are centered exclusively on a simplified version of Becker's (1968) model.

Although early economists were aware of the importance of theft as an allocation process, not until the 1960s did economists begin to formalize crime as an economic activity performed by rational agents. Since Becker's (1968) seminal paper, several strands of literature that adopt the economic approach to the study of crime have developed. Early papers use models in which individuals react to incentives and the aggregate behavior is made consistent through the adjustment of the relevant endogenous variables. See Ehrlich (1996) for an overview. Other papers adopt a search-theoretic approach to model an economy with theft, prominent examples

Machin, and Witt (2011), Chalfin and McCrary (2018), and Weisburd (2021). See also Levitt and Miles (2006) and Chalfin and McCrary (2017) for additional references.

being Burdett, Lagos, and Wright (2003, 2004). Finally, there are a few papers that introduce theft in a Walrasian model. Important representatives of this literature are Usher (1987), Grossman (1994), and Dal Bó and Dal Bó (2011). A noteworthy attribute of the above models is that they predict in some way or another, that law enforcement unequivocally deters crime. The models that follow Becker's approach assume that the supply of criminal offenses is negatively related to the probability of apprehension. The search model proposed by Burdett, Lagos, and Wright (2003) exhibits multiple types of equilibria, in all of which police reduce crime. The same prediction arises from the model in Dal Bó and Dal Bó (2011). The reason is that in these models, although the relevant endogenous variables affect the level of crime, the level of crime does not have any feedback effect on the other variables.

In a recent paper, Lasso de la Vega, Volij, and Weinschelbaum (2021) introduce theft into a Walrasian general equilibrium model and analyze, among other things, the effect of law enforcement on crime. They consider two scenarios that differ on the wealth that is subject to theft. In the first scenario, all factors of production are subject to theft, and in the other one, only produced goods are. They found that whereas in the first scenario, more police unequivocally reduce crime, this is not the case in the second one. Specifically, they give an example in which an increase in police increases crime. The reason is that whereas more police initially has a negative incentive on thieves, it also promotes economic activity which in turn makes theft more profitable.

We should stress that this result is obtained within a textbook general equilibrium model and therefore one cannot suspect that the unusual result stems from a contrived ad-hoc model. Still, one may wonder whether the perverse effect of police on crime is due to unrealistic primitives of the economy that lead to a pathological example. In this paper we show that this is not the case. Specifically, we show that for any set of values for the endogenous variables, we can calibrate an economy that fit them and where more police induces more crime. One may object that the calibrated economy exhibits a very special appropriation technology. However, we also show that for any appropriation technology that satisfies a minimal condition, one can build an economy with a Cobb-Douglas production function and a linear demand function such that an increase in police protection induces an increase in crime.

One may object that once a model allows for general equilibrium effects anything can happen. In particular, it may not be that surprising that in such a model police has a perverse effect on crime. We show, however, that even accounting for feedback effects of crime on markets, whereas an increase in police spending may reduce property crime, it unequivocally decreases the share of the GDP that is ultimately stolen.

One may still wonder whether the perverse effect of police on crime stems from a suboptimal level of police protection. For instance, Chalfin and McCrary (2018) suggest that "additional investments in police are unlikely to be socially beneficial unless police reduce violent crimes to at least a moderate degree." Indeed, it would seem puzzling if reducing police spending resulted in a reduction in crime and yet it was best not to apply such a policy. We show, however, that it may well be the case that at the optimal level of police protection police has a perverse effect on crime. The reason is that one of the consequences of lower police protection is a decrease in output whose social cost may outweigh the benefits from lower crime and police.

The rest of the paper is organized as follows. Section 2 introduces the model and characterizes its competitive equilibrium. Section 3 establishes our main results.

2 The model

We now present a version of the general equilibrium model with theft introduced by Lasso de la Vega, Volij, and Weinschelbaum (2021), under the assumption that only produced goods are subject to theft. The primitives of the model are the following. There is a publically available technology that transforms capital and labor into a consumption good, which will be henceforth referred to as *peanuts*. This technology is described by a constant returns to scale, monotone and concave production function $F(K, L)$. There is a continuum of individuals $I = [0, 1]$, characterized by a quasilinear utility function $u_i(x, \ell) = \phi_i(x) + \ell$, and initial endowment of capital $\overline{k}_i$ and labor $\bar{l}_i$. Further, we assume that ϕ_i is strictly increasing, concave, and that $\lim_{x\to\infty} \phi_i'(x) = 0$. For notational convenience we will assume that all individual are identical, namely $\phi_i = \phi$, $\overline{k}_i = \overline{K}$ and $\bar{l}_i = \overline{L}$ for all $i \in [0, 1]$. Furthermore, to avoid dealing with boundary problems, we assume that individuals can consume negative amounts of leisure and that the production function satisfy the Inada conditions. In particular, $\lim_{L\to 0} F_2(K, L) = \infty$.[2]

There is an appropriation sector that uses labor to redistriture output from earners to thieves. We follow Grossman (1994) and Dal Bó and Dal Bó (2011), and describe the appropriation technology by a function $A : \mathbb{R}^2_+ \to [0, 1]$. The value $A(Y, T)$ is the proportion of the individuals income that gets stolen when the crime level is Y and police protection T. We call $A(Y, T)$ the *excise rate of theft* associated with Y and T. We assume that $A(0, T) = 0$, that A is increasing and strictly concave in its first argument, decreasing and strictly convex in its second argument, and that $A_{12} < 0$, namely the marginal excise rate of crime is decreasing in police protection.These assumptions imply that

$$A_1(Y, T) < \frac{A(Y, T)}{Y}$$

and that $\lim_{Y\to 0} A(Y, T)/Y = A_1(0, T)$. Namely, the marginal excise rate is lower than the average excise rate. We denote by $a(Y, T)$ the average excise rate, with the extension $a(0, T) = A_1(0, T)$. It is the proportion of wealth stolen per unit of time devoted to theft. It follows from our assumptions that $a(Y, T)$ is decreasing in both its arguments, and convex in its second argument. We summarize the data of the economy by $\mathcal{E} = \langle(\phi, \overline{K}, \overline{L},), F, A, T\rangle$.

[2]For any function $f : \mathbb{R}^2 \to \mathbb{R}$, we denote by f_1 and f_2 its partial derivatives with respect to its first and second arguments. Also f_{jk}, for $j, k = 1, 2$, stand for the corresponding second derivatives.

Individuals, apart from consuming peanuts and leisure, devote some time to theft. A *bundle* for individual i is thus a triple $(x_i, \ell_i, y_i) \in \mathbb{R}_+ \times \mathbb{R} \times \mathbb{R}_+$ whose components are the amounts of peanuts, leisure, and time devoted to theft. We denote the set of bundles by $\mathcal{X}$.

When individual i devotes y_i units of his time to theft, the crime rate is $Y = \int_0^1 y_i di$, and he gets a portion y_i/Y of the booty.[3] There is a level T of public police protection which is allocated uniformly across individuals and is financed by means of compulsory taxation.

An *allocation* in $\mathcal{E}$ consists of an input pair $(K, L) \in \mathbb{R}^2_+$, an assignment $(x, \ell, y) : [0, 1] \to \mathcal{X}$ of bundles to individuals, and a crime rate Y. An allocation is *feasible* if

$$\begin{aligned} \int x &= F(K, L) \\ \overline{L} &= \int \ell + L + \int y + T \\ \overline{K} &= K \\ Y &= \int y. \end{aligned} \tag{1}$$

Namely, peanuts consumed are equal to peanuts produced, the sum of time devoted to leisure, labor, theft and police protection equals the total time available, capital used in the production process equals the amount of capital available, and the crime rate is the per capita time devoted to theft.

2.1 Competitive equilibrium

We normalize the wage rate to be 1, and for simplicity, we assume that public police is financed by uniform taxation. The resources that an individual has available for the purchase of peanuts consist of the portion of his legitimate income (net of taxes) that is not stolen, plus the proceeds from his appropriation activities. Under our

[3] For any real function f defined on $[0, 1]$, we will sometimes write $\int f$ for $\int_0^1 f_i \, di$. All functions defined on $[0, 1]$ are assumed to be integrable.

assumption that only produced goods are subject to theft, the returns to theft are given by $a(Y,T)pF(\overline{K},L)$ and therefore, an individual's budget is given by

$$B = \left\{(x_i,\ell_i,y_i) : px_i \leq (1-A(Y,T))(r\overline{K}+\overline{L}-\ell_i-y_i-T)+y_i a(Y,T)pF(\overline{K},L)\right\}$$

The parameters that the individual takes as given are the price of peanuts p, the rental rate of capital r, the tax T, the crime rate Y, and the returns to theft.[4] Note that $\overline{L}-\ell_i-y_i-T$ is the time that individual i devotes to labor. Also note that the relative price of peanuts (in terms of leisure) faced by the consumers is $p/(1-A(Y,T))$. This is so because for every unit of time that they devote to work, $A(Y,T)$ is stolen and therefore only $(1-A(Y,T))$ can be used to purchase peanuts. Equivalently, a consumer who wants to bring home one unit of peanuts needs to buy $1/(1-A(Y,T))$ units because a proportion $A(Y,T)$ of them will be stolen.

The concept of competitive equilibrium is the usual one.

Definition 1 A *competitive equilibrium* consists of a price of peanuts p^*, a rental rate of capital r^* and a feasible allocation $\langle (L^*,K^*),(x^*,\ell^*,y^*),Y^*\rangle$, such that

1. The input pair (L^*,K^*) maximizes profits given p^* and r^*.

2. For all $i\in[0,1]$, the bundle (x_i^*,ℓ_i^*,y_i^*) maximizes the individual's utility given p^*, r^* and Y^*.

2.2 Characterization of the equilibrium

Given our assumptions on preferences and technology any equilibrium allocation $\langle (L^*,K^*),(x^*,\ell^*,y^*),Y^*\rangle$ must satisfy $K^*>0$, $L^*>0$ and $x>0$. Therefore, the necessary (and sufficient) conditions for profit maximization are:

$$\begin{aligned} p^*\frac{\partial F}{\partial L}(K^*,L^*) &= 1 \\ p^*\frac{\partial F}{\partial K}(K^*,L^*) &= r^* \end{aligned}$$

[4] We ignore his share in the firms' profits since, given the constant returns to scale technology, profits will be 0 in equilibrium.

Namely, input prices must be equal to the value of their marginal productivity.

Given than in equilibrium, the capital used by the firms, K^*, must be $\overline{K}$, it is convenient to define the firm's short-run supply function. It is the function $Q : \mathbb{R}_{++} \to \mathbb{R}_+$ implicitly defined by

$$\begin{aligned} 1 &= p\frac{\partial F}{\partial L}(\overline{K}, L) && (2) \\ Q(p) &= F(\overline{K}, L) && (3) \end{aligned}$$

The first-order conditions for individual i's utility maximization are:

$$\begin{aligned} \phi'(x_i^*) &= \frac{p^*}{1 - A(Y^*, T)} && (4) \\ 1 - A(Y^*, T) &\geq a(Y^*, T)p^*F(K^*, L^*) \quad \text{with equality if } y_i^* > 0 && (5) \\ p^*x_i^* &= (1 - A(Y^*, T))(r^*K^* + L) + y_i^*a(Y^*, T)pF(K^*, L^*) \end{aligned}$$

where Y^* is the crime rate associated with the equilibrium allocation. Observe that in equilibrium $A(Y^*, T) < 1$, which follows from (4). Condition (5) is an arbitrage condition which states that the returns to theft cannot exceed the returns to labor, and they must be equal if the individual devotes positive time to theft. It will be convenient to define the aggregate demand function for peanuts. It is the function $X : \mathbb{R}_{++} \to \mathbb{R}_+$ implicitly defined by

$$\phi'(X(p)) = p.$$

Finally, the allocation must satisfy the feasibility conditions (1).

Upon close observation of the above conditions, and taking advantage of the definitions of the short-run aggregate supply and demand functions, we can see that to find an equilibrium it is enough to solve

$$\begin{aligned} 1 - A(Y, T) &\geq a(Y, T)pQ(p) \quad \text{with equality if } Y > 0 && (6) \\ X\left(\frac{p}{1 - A(Y, T)}\right) &= Q(p) && (7) \end{aligned}$$

This is a system of two equations with two unknowns (p and Y). Once solved, the other variables are obtained by mere substitution. Indeed, the remaining variables,

L^*, r^*, and ℓ^* are directly obtained from

$$\begin{aligned} F(\overline{K}, L^*) &= Q(p^*) \\ p^* \frac{\partial F}{\partial K}(\overline{K}, L^*) &= r^* \\ \overline{L} - T - Y^* &= \int \ell^*. \end{aligned}$$

Note that in equilibrium

$$Y^* = \frac{A(Y^*, T)}{1 - A(Y^*, T)} p^* Q(p^*). \tag{8}$$

Indeed, if $Y^* = 0$, this equality is trivially satisfied. And if $Y^* > 0$, it follows from (6). Recall that $p^*/(1 - A(Y^*, T))$ is the peanut price faced by the consumers. Therefore, the above equation says that in equilibrium, the aggregate time devoted to theft equals the value of the stolen goods at consumer prices. For that reason, it is justified to call Y^* the level of theft or of (property) crime. Note that

$$\frac{A(Y^*, T)}{1 - A(Y^*, T)} p^* Q(p^*) = A(Y^*, T) p^* Q(p^*) + A^2(Y^*, T) p^* Q(p^*) + \cdots$$

That is, property crime at the equilibrium crime rate Y^* is not just the proportion $A(Y^*, T)$ of the GDP. The portion $A(Y^*, T)p^*Q(p^*)$ is only the peanuts stolen from the income legitimately earned by the agents. But property crime includes also the peanuts stolen from the stolen income as well.

Figure 1 depicts the equilibrium in the peanut market (where $A(Y^*, T)$ is denoted simply by A^*).

The price faced by the consumers is $p^*/(1 - A(Y^*, T))$ and the price faced by the firm is p^*. The difference is $p^* \frac{A(Y^*, T)}{1 - A(Y^*, T)}$. As can be seen, theft has a similar effect to that of an ad valorem tax of $A(Y^*, T)/(1 - A(Y^*, T))$. It introduces a wedge between the effective price paid by the consumers and the one received by the firm. The difference is the value of the peanuts being stolen when one acquires one unit of peanuts. However, since by (8), the value of the stolen peanuts equals the value of the time spent on appropriation activities, this value ultimately dissipates.

Lasso de la Vega, Volij, and Weinschelbaum (2021) shows that there are economies with no equilibrium. However, it also shows that if the appropriation

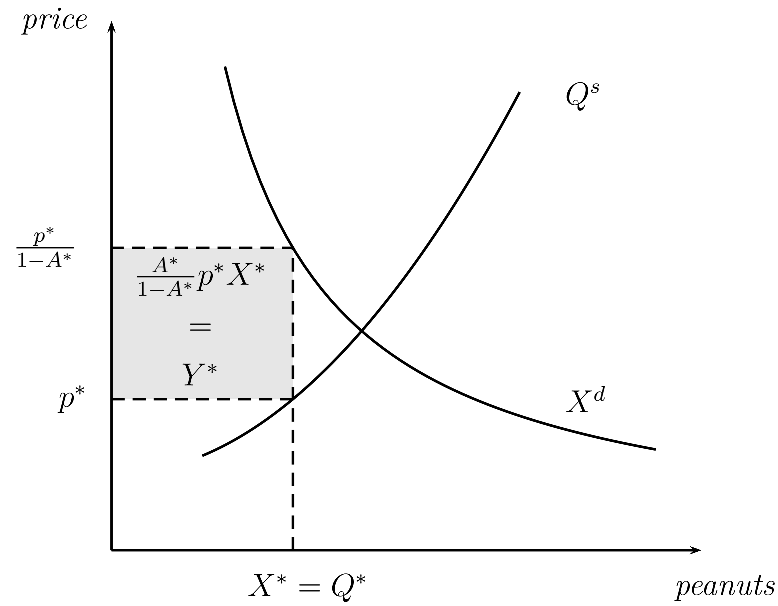

Figure 1: The peanut market.

technology satisfies certain conditions, an equilibrium exists and is unique. This is stated in the following observation. Since it is short, we include a proof that fits this version of the model.

Observation 1 Let $\mathcal{E} = \langle(\phi, \overline{K}, \overline{L},), F, A, T\rangle$ be an economy. Then, if the appropriation technology A is bounded away from one, an equilibrium exists. If, furthermore, $\frac{a(Y,T)}{1-A(Y,T)}$ is non-increasing in Y, the equilibrium is unique.

Proof : See Appendix. □

Appropriation technologies A that are bounded away from one and such that $\frac{a(Y,T)}{1-A(Y,T)}$ is non-increasing in Y are said to be *regular*. For future reference, we define $g(Y,T) = \frac{a(Y,T)}{1-A(Y,T)}$. Given our assumptions on A, the function g satisfies $g_2 < 0$, and whenever A is regular, $g_1 \leq 0$.

3 The effect of police on crime

We want to focus on the effect of police protection on the crime rate. It can be seen from equations (6–7) that the peanut market affects the crime rate and simultaneously the crime rate affects the peanut market. For that reason, the effect of changes in police protection on crime is ambiguous, as we will see later. The following example shows that an increase in police protection may well increase crime.

Example 1 Consider the economy $\mathcal{E} = \left\langle(\phi, \overline{K}, \overline{L}), F, A, T\right\rangle$ where $\phi(x) = x(9 - x/2)$, $\overline{K} = 1$, $F(K, L) = \sqrt{KL}$, and $A(Y, T) = \frac{3}{4}\frac{Y}{1+Y+2\frac{T}{1+T}}$. The aggregate demand function is given by $X(p) = 9 - p$ and the short-run supply function is given by $Q(p) = p/2$. The unique equilibrium of this economy results from the solution of equations (6-7), which is

$$Y(T) = \frac{5 - 3T}{3T + 3} + \sqrt{\frac{57T + 25}{T + 1}} \qquad p(T) = 1 + \frac{1}{3}\sqrt{\frac{57T + 25}{T + 1}}.$$

The equilibrium crime rate $Y(T)$ is plotted in Figure 2. It can be seen that for $T < 11/21$, the crime rate increases with T. In particular, the maximum crime rate is not attained by reducing police funding to 0.

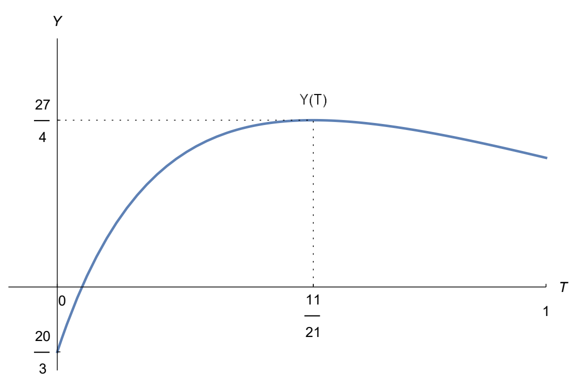

Figure 2: The equilibrium crime rate as a function of police protection.

In what follows we will show that the above example is robust. Specifically, we will show that for any level of police protection and for any positive values of crime rate, price and output, an economy can be calibrated to fit these values and in which an increase in police results in an increase in crime.

Before that, we show a useful lemma. Recall that $g(Y,T) = \frac{a(Y,T)}{1-A(Y,T)}$ and notice that any equilibrium of an economy $\mathcal{E} = \langle(\phi, \overline{K}, \overline{L},), F, A, T_0\rangle$ with positive crime rate Y^* and peanut price p^* is characterized by

$$1 = g(Y, T_0) p Q(p) \qquad (9)$$

$$X\Big(\frac{p}{1 - A(Y, T_0)}\Big) = Q(p). \qquad (10)$$

These equations implicitly define the equilibrium crime rate $Y(T)$ and peanut price $p(T)$ as functions of police protection in a neighborhood of T_0, with $Y(T_0) = Y^*$ and $p(T_0) = p^*$. The following lemma establishes conditions for such an equilibrium to exhibit a perverse effect of police on crime.

Lemma 1 If the equilibrium crime rate Y^* is positive, and $g_1(Y^*, T_0) \leq 0$, then $Y'(T_0) > 0$ if and only if

$$\frac{Q'(p^*) - X'(\frac{p^*}{1-A(Y^*,T_0)})}{Q'(p^*) - X'(\frac{p^*}{1-A(Y^*,T_0)})(1+\eta)} < A(Y^*, T_0) \qquad (11)$$

where η is the elasticity of the supply function $Q(p)$ at p^*.

Proof : See Appendix. □

We are now ready to state our first result.

Theorem 1 Let T_0 be a given level of public police protection. Let $p^* > 0$, $Q^* > 0$, and $Y^* > 0$, be a price, quantity of peanuts, and a crime rate. We can find an economy $\mathcal{E}_0 = \langle(\phi, \overline{K}, \overline{L}), F, A, T_0\rangle$ such that at its unique equilibrium the price, output and crime rate are given by p^*, Q^*, and Y^*, respectively, and such that a small increase in police protection results in an increase in crime.

Proof : We will build an economy $\mathcal{E}_0 = \langle(\phi, \overline{K}, \overline{L}), F, A, T_0\rangle$ with peanut price p^*, output Q^*, equilibrium crime rate $Y^* > 0$, and such that if police protection is slightly increased the crime rate will also increase. Lemma 1 establishes conditions for this to occur. We now build the economy $\mathcal{E}_0 = \langle(\phi, \overline{K}, \overline{L}), F, A, T_0\rangle$, that satisfies them.

If p^*, Q^*, and Y^* are the equilibrium price, output, and crime rate of some economy with police protection given by T_0, using equation (8) we obtain that the equilibrium excise rate of theft is given by,

$$A(Y^*, T_0) = \frac{Y^*}{p^*Q^* + Y^*}.$$

Let $0 < \alpha < A(Y^*, T_0)$. This α can be found since $Y^* > 0$. The production function of the economy is $F(K, L) = K^\alpha L^{1-\alpha}$. Therefore, when the capital level is fixed at K, the corresponding short-run supply function is $Q(p) = K((1-\alpha)p)^{\frac{1-\alpha}{\alpha}}$. Note that the elasticity of supply is $\eta = \frac{1-\alpha}{\alpha}$. We now choose the capital endowment to be $\overline{K}$ such that $Q(p^*) = Q^*$. The choice of $\overline{L}$ is arbitrary although it can be chosen to be large enough so that the resulting equilibrium leisure is positive.

We now choose the utility function. Let $\hat{b} > 0$ and $\hat{c} > 0$ be the unique solution of

$$\frac{Q'(p^*) + b/2}{Q'(p^*) + b(1+\eta)/2} = A(Y^*, T_0) \qquad (12)$$

$$c - b\frac{p^*}{1 - A(Y^*, T_0)} = Q^* \qquad (13)$$

These $\hat{b}$ and $\hat{c}$ can be found because the left-hand side of (12) equals 1 when $b = 0$ and is decreasing in b with

$$\lim_{b\to\infty} \frac{Q'(p^*) + b/2}{Q'(p^*) + b(1+\eta)/2} = 1/(1+\eta) = \alpha < A(Y^*, T_0).$$

Note that the choice of $\hat{b}$ implies that

$$\frac{Q'(p^*) + \hat{b}}{Q'(p^*) + \hat{b}(1+\eta)} < A(Y^*, T_0). \qquad (14)$$

We choose the consumers' utility function to be $\phi(x) = \frac{x(\hat{c} - x/2)}{\hat{b}}$. Consequently, the corresponding demand function is $X(p) = \hat{c} - \hat{b}p$, which implies that $X'(p) = -\hat{b}$.

It remains to choose the appropriation technology. It will be given by

$$A(Y,T) = \frac{dY}{Y + p^*Q^*(\frac{T}{1+T} + e)}$$

where $e = \frac{1}{2(1+T_0)}$ and $d = \frac{\frac{p^*Q^*}{2} + p^*Q^*T_0 + T_0Y^* + Y^*}{p^*Q^* + p^*Q^*T_0 + T_0Y^* + Y^*}$. It is routine to check that $1 = g(Y^*, T_0)p^*Q^*$. Also, by the choice of the utility function (see equation (13)), $X(\frac{p^*}{1-A(Y^*,T_0)}) = Q^*$. This means that the economy $\mathcal{E}_0 = \langle(\phi, \overline{K}, \overline{L}), F, A, T_0\rangle$ has an equilibrium with price, quantity and crime rate given by p^*, Q^*, and Y^*. Furthermore, since $d < 1$, $A(Y,T)$ is bounded away from one. Also, it can be checked that $g_1(Y,T) < 0$. Hence, the appropriation technology is regular and, by Observation 1, the equilibrium is unique. By construction, inequality (14) holds which, since $b = -X'(p)$, implies that inequality (11) holds. By Lemma 1 we conclude that an increase in police protection, increases the crime rate. □

One may argue that the perverse effect of police on crime identified in the above theorem results from a very peculiar appropriation technology. The next theorem, however, shows that this kind of perverse effect is compatible with any appropriation technology that is bounded away from one.

Theorem 2 Let T_0 be a given level of public police protection. Let $A : \mathbb{R}^2_+ \to [0,1)$ be an appropriation technology that is bounded away from one. Then there is an economy with appropriation technology A and police protection T_0 such that in equilibrium a small increase in police results in an increase in crime.

Proof : We will build an economy $\mathcal{E}_0 = \langle(\phi, \overline{K}, \overline{L}), F, A, T_0\rangle$ with a positive equilibrium crime rate $Y^* > 0$ and equilibrium price p^*, such that if police protection is slightly increased the crime rate will also increase. Lemma 1 establishes conditions for this to occur. Our task is then, to build an economy that satisfies them.

Let $\alpha \in (0,1)$ such that $A(Y,T_0) > \alpha$ for some Y, let $F(K,L) = K^\alpha L^{1-\alpha}$, and let $\overline{K} = 1$. As a result, the associated short-run aggregate supply function is given by $Q(p) = \big((1-\alpha)p\big)^{\frac{1-\alpha}{\alpha}}$ whose elasticity is $\eta = (1-\alpha)/\alpha$.

Since A is bounded away from one, we have that $\lim_{Y\to\infty} g(Y,T) = 0$. Therefore, for any Y, there is $\hat{Y} \geq Y$ such that $g_1(\hat{Y}, T_0) < 0$. Consequently, we can choose $Y^* > 0$ such that both $A(Y^*, T_0) > \alpha$ and $g_1(Y^*, T_0) < 0$. This Y^* will be the equilibrium crime rate in the economy we are looking for.

For any $c > 0$, consider the utility function given by $\phi_c(x) = x(c - x/2)$. The associated demand function is $X_c(p) = c - p$. The peanut market clearing condition $X_c(\frac{p}{1-A(Y^*,T_0)}) = Q(p)$ can be written as

$$c - \frac{p}{1 - A(Y^*, T_0)} = Q(p)$$

It can be checked that this equation has a unique solution. Denote it by $p(c)$ and notice that $\partial p/\partial c > 0$ and that $\lim_{c\to\infty} p(c) = \infty$.

Consider now the function

$$f(c) = g(Y^*, T_0)p(c)Q(p(c))$$

We have that f is increasing in c, $f(0) = 0$ and $\lim_{c\to\infty} f(c) = \infty$. By the intermediate value theorem, there is $\hat{c}$ such that $f(\hat{c}) = 1$. This means that $p^* = p(\hat{c})$ and Y^* satisfy

$$1 = g(Y^*, T_0)p^*Q(p^*) \tag{15}$$

$$X_{\hat{c}}\left(\frac{p^*}{1 - A(Y^*, T_0)}\right) = Q(p^*) \tag{16}$$

In other words, p^* and Y^* are equilibrium price and crime rate of the economy $\mathcal{E}_{\hat{c}} = \left\langle (\phi_{\hat{c}}, \overline{K}, \overline{L}), F, A, T_0 \right\rangle$, where $\overline{L}$ is arbitrary but can be chosen so that the equilibrium per capita leisure is positive.

We have built an auxiliary economy with an equilibrium price p^* and crime rate Y^*. However, this equilibrium does not necessarily satisfy the conditions of Lemma 1. We now build a collection of economies with the same equilibrium price and crime rate and show that one of them satisfies these conditions.

For any $b > 0$, let

$$c(b) = Q(p^*) + b\frac{p^*}{1 - A(Y^*, T_0)}$$

and define the following utility function: $\phi^b(x) = \frac{x(c(b)-x/2)}{b}$. Note that the corresponding demand function is $X^b(p) = c(b) - bp$. Let $\mathcal{E}^b = \langle(\phi^b, \overline{K}, \overline{L}), F, A, T_0\rangle$ be the economy that is obtained from $\mathcal{E}_{\hat{c}}$ by replacing the utility function $\phi_{\hat{c}}$ by ϕ^b. It can be checked that p^* and Y^* are equilibrium values of both the economies $\mathcal{E}^b$ and $\mathcal{E}_{\hat{c}}$. Indeed, if we substitute p^* for p and Y^* for Y in $\mathcal{E}^b$'s equilibrium conditions

$$1 = g(Y, T_0)pQ(p) \tag{17}$$

$$c(b) - b\frac{p}{1 - A(Y, T_0)} = Q(p) \tag{18}$$

and compare them with equations (15–16) we see that these conditions hold. See Figure 3.

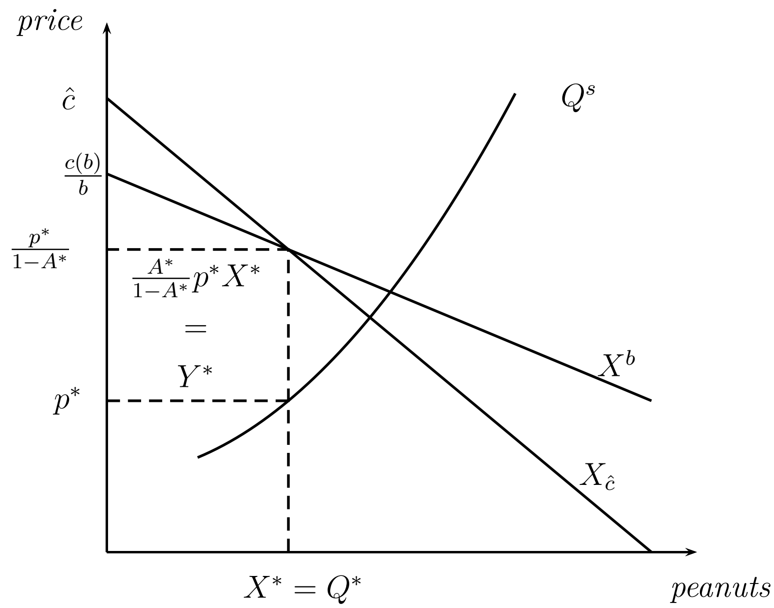

Figure 3: The pivoted demand function.

We now single out the economy $\mathcal{E}_0$ alluded to in the statement of the theorem. Since p^* does not depend on b and since $\eta = \frac{1-\alpha}{\alpha}$,

$$\lim_{b\to\infty} \frac{Q'(p^*) + b}{Q'(p^*) + b(1 + \eta)} = \frac{1}{1 + \eta} = \alpha < A(Y^*, T_0).$$

Therefore, we can find $\tilde{b}$ large enough so that

$$\frac{Q'(p^*) + \tilde{b}}{Q'(p^*) + \tilde{b}(1+\eta)} < A(Y^*, T_0).$$

Given that $\tilde{b} = -(X^{\tilde{b}})'(\frac{p^*}{1-A(Y^*,T_0)})$ and that $g_1(Y^*, T_0) < 0$, this shows that the economy $\mathcal{E}^{\tilde{b}}$ satisfies condition (11). Therefore, by Lemma 1, $Y'(T_0) > 0$ and $\mathcal{E}^{\tilde{b}}$ is the economy $\mathcal{E}_0$ that we were looking for. □

Theorem 2 shows that when theft is introduced into a general equilibrium model, the effect of police on the crime rate is ambiguous. However, in economies with regular appropriation technologies, even accounting for general equilibrium effects, increases in police protection unambiguously reduce the excise rate of theft and increase the level of output. This is stated in the following proposition.

Proposition 1 Let $\mathcal{E} = \langle(\phi, \overline{K}, \overline{L}), F, A, T\rangle$ be an economy with a regular appropriation technology. Assume that the equilibrium crime rate is positive. Then, the associated equilibrium peanut price $p(T)$ and output, $Q(p(T))$ are increasing, and the excise rate, $A(Y(T), T)$ is decreasing in the level of police protection.

Proof : By Observation 1, $\mathcal{E}$ has a unique equilibrium, which is characterized by equations (6)-(7). These equations implicitly define the equilibrium price $p(T)$ and crime rate $Y(T)$. By the implicity function theorem, we obtain

$$p'(T) = \frac{pQX'\left(A_2 g_1 - A_1 g_2\right)}{X'\left(gA_1\left(pQ' + Q\right) - (1-A)Qg_1\right) + (1-A)^2 Qg_1Q'} \tag{19}$$

$$Y'(T) = -\frac{X'\left(gA_2\left(pQ' + Q\right) - (1-A)Qg_2\right) + (1-A)^2 Qg_2Q'}{X'\left(gA_1\left(pQ' + Q\right) - (1-A)Qg_1\right) + (1-A)^2 Qg_1Q'} \tag{20}$$

where we have used the following simplifying notation: $p = p(T)$, $g = g(Y(T), T)$, $g_2 = g_2(Y(T), T)$, $g_1 = g_1(Y(T), T)$, $A = A(Y(T), T)$, $A_2 = A_2(Y(T), T)$, $A_1 = A_1(Y(T), T)$, $Q = Q(p(T))$, $Q' = Q'(p(T))$, and $X' = X'(\frac{p(T)}{1-A(Y(T),T)})$. Given our assumptions on the appropriation technology, we see that $p'(T) > 0$. As a result, since $Q'(p) > 0$, we also obtain that the equilibrium level of output is increasing in T. Finally, we have

$$\begin{aligned}\frac{dA}{dT} &= A_1 Y'(T) + A_2 \\ &= \frac{(1-A)Q\,(A_2 g_1 - A_1 g_2)\,((1-A)Q' - X')}{X'\,(gA_1\,(pQ'+Q) - (1-A)Qg_1) + (1-A)^2 Q g_1 Q'}\end{aligned}$$

which, given the properties of g and A, can be checked to be negative. □

3.1 Optimal police

In this section we characterize the optimal level of police protection and show that even when police protection is set at the optimal level, it may well be the case that more police induces more crime.

Let $\mathcal{E} = \left\langle(\phi, \overline{K}, \overline{L}), F, A, T\right\rangle$ be an economy with a regular appropriation technology, and let $Y(T)$ the equilibrium crime rate, which is assumed to be positive, and let $p(T)$ the equilibrium price. Denote by $X^*(T)$ and $Q^*(T)$ the equilibrium quantity of peanuts demanded and produced. Namely, $X^*(T) = X(p(T))$, and $Q^*(T) = Q(p(T))$. Given that preferences are quasilinear, we can evaluate the social desirability of allocations by the associated utility they generate. Thus, the social welfare corresponding to the equilibrium allocation when police protection is T, is given by

$$W(T) = \phi(X^*(T)) + \overline{L} - c(Q^*(T)) - Y(T) - T$$

where c is the short-run cost function associated with the production function F. Namely, $c : \mathbb{R}_+ \to \mathbb{R}_+$ is implicitly defined by $Q = F(\overline{K}, c(Q))$. The optimal level of public police protection satisfies

$$\phi'(X^*(T))X^{*\prime}(T) = c'(Q^*(T))Q^{*\prime}(T) + Y'(T) + 1.$$

The optimal level of police protection equalizes the marginal cost of police with its marginal benefit. The marginal cost consists of three components: the production cost of the additional output induced by the additional police, the increase (which may be negative) in the crime rate, and the additional expenditure on police. The marginal benefit is the increase in the consumers' utility due to the additional consumption of peanuts.

Since in equilibrium $X^*(T) = Q^*(T)$, $\phi'(X^*(T)) = p(T)/(1 - A(Y(T),T))$ and $c'(Q^*(T)) = p^*(T)$, we have that the optimal level of public police protection satisfies

$$\frac{A(Y(T),T)}{1 - A(Y(T),T)} p(T) Q^{*\prime}(T) = 1 + Y'(T). \tag{21}$$

By Proposition 1, we have that the left-hand side of this equation is positive. Therefore, it may well be the case that even at the optimal level of police protection we have that $Y'(T) > 0$, as the following example illustrates.

Example 1 (cont.): Recall that in the economy of Example 1, the level of crime is increasing in police protection as long as $T < 11/21$. Figure 4 depicts the equilibrium social welfare as a function of police protection. It can be seen that it attains its maximum at $T^* = 0.419 < 11/21$. Therefore, in this economy, even at the optimal level of public police protection, an increase in police induces an increase in crime.

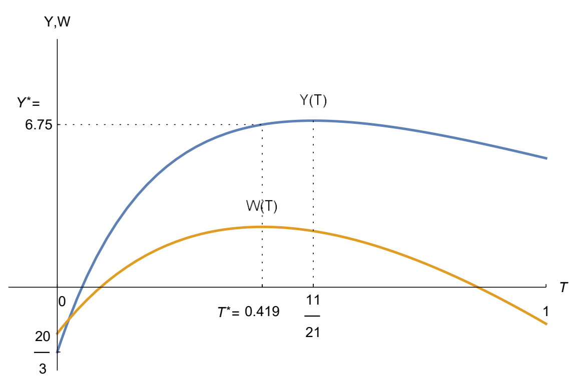

Figure 4: The optimal level of police protection.

One would wonder why the social planner would not want to reduce police protection, even when at the optimum such reduction would induce a decrease in crime. The reason can be seen in equation (21): a reduction in police protection decreases the equilibrium output with a corresponding reduction in consumer surplus, which turns out not to be compensated by the savings in police expenditure and the reduction in crime.

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A Appendix

Proof of Observation 1 By our assumptions on ϕ and F, for any fixed Y, equation (7) has a unique solution, which we denote by $p(Y)$. It can be checked that $p(Y)$ is non-increasing in Y and, furthermore, that $Q(p(Y))$ is decreasing in Y. Therefore, $\mathcal{E}$ has an equilibrium if

$$1 - A(Y,T) \geq a(Y,T)p(Y)Q(p(Y)) \quad \text{with equality if } Y > 0.$$

Since A is bounded away from one, this is equivalent to

$$1 \geq \frac{a(Y,T)}{1 - A(Y,T)} p(Y)Q(p(Y)) \quad \text{with equality if } Y > 0. \tag{22}$$

If $1 \geq \frac{a(0,T)}{1-A(0,T)}p(0)Q(p(0))$, then $Y^* = 0$ solves (22). If, on the other hand, $1 < \frac{a(0,T)}{1-A(0,T)}p(0)Q(p(0))$, then, given that $p(Y)Q(p(Y))$ is decreasing, that A is bounded away from one, and that $a(Y,T)$ goes to 0 as Y goes to ∞, we have that $\frac{a(Y,T)}{1-A(Y,T)}p(Y)Q(p(Y)) \to 0$ as Y goes to ∞. By the intermediate value theorem, there is Y^* such that $1 = \frac{a(Y^*,T)}{1-A(Y^*,T)}p(Y^*)Q(p(Y^*))$ and an equilibrium exists. If $\frac{a(Y,T)}{1-A(Y,T)}$ is non-increasing, this Y^* is unique and so is the equilibrium. $\square$

Proof of Lemma 1 As mentioned before, any equilibrium of $\mathcal{E}_0$ with positive crime rate is characterized by equations (9–10). By the implicit function theorem

$$Y'(T_0) = -\frac{X'gA_2\left(p^*Q' + Q\right) + (1-A)g_2Q\left((1-A)Q' - X'\right)}{X'gA_1\left(p^*Q' + Q\right) + (1-A)g_1Q\left((1-A)Q' - X'\right)}$$

where we have used the following simplifying notation: $g = g(Y^*, T_0)$, $g_1 = g_1(Y^*, T_0)$, $g_2 = g_2(Y^*, T_0)$, $A = A(Y^*, T_0)$, $A_1 = A_1(Y^*, T_0)$, $A_2 = A_2(Y^*, T_0)$, $Q = Q(p^*)$, $Q' = Q'(p^*)$, and $X' = X'(\frac{p^*}{1-A(Y^*,T_0)})$. Recall that in equilibrium $A(Y^*, T_0) < 1$ and hence these values are well defined. Given our assumption that $g_1(Y^*, T_0) < 0$, we have that the denominator of the above expression is negative.

As a result, $Y'(T_0) > 0$ if and only if the numerator is positive, namely

$$\begin{aligned}
Y'(T_0) > 0 \Leftrightarrow \quad & -X' g A_2 \left(p^* Q' + Q\right) && < && (1-A) g_2 Q \left((1-A)Q' - X'\right) \\
\Leftrightarrow \quad & -X' \frac{A}{Y^*(1-A)} A_2 \left(p^* Q' + Q\right) && < && (1-A) \frac{A_2}{Y^*(1-A)^2} Q \left((1-A)Q' - X'\right) \\
\Leftrightarrow \quad & -X' A A_2 \left(p^* Q' + Q\right) && < && A_2 Q \left((1-A)Q' - X'\right) \\
\Leftrightarrow \quad & -X' A \left(p^* Q' + Q\right) && > && Q \left((1-A)Q' - X'\right)
\end{aligned}$$

where we have used the fact that $g_2 = A_2/(Y^*(1-A)^2)$ and that $A_2 < 0$. Dividing both sides by $Q(p^*)$ we obtain that $Y'(T_0) > 0$ if and only if

$$-X' A \left(1 + \eta\right) > \left((1-A)Q' - X'\right).$$

Isolating A and returning to the more detailed notation, we conclude that $Y'(T_0) > 0$ if and only if

$$\frac{Q'(p^*) - X'(\frac{p^*}{1-A(Y^*,T_0)})}{Q'(p^*) - X'(\frac{p^*}{1-A(Y^*,T_0)})(1+\eta)} < A(Y^*, T_0).$$

□